Breaking the Plane: An Exploration of 3D AR Visualization in Augmenting Comprehension of Multidimensional Mathematical Concepts

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We introduce Breaking the Plane, an augmented reality (AR) application built for the Meta Quest 3 headset that enables users to visualize 3D mathematical functions using handwritten input. Researchers have demonstrated that AR increases motivation to learn mathematics and enhances mathematical learning, while also revealing that optical character recognition (OCR) makes the authoring of teaching materials more accessible and time-efficient for instructors. Previous work has developed AR systems that separately employ OCR and 3D AR, but work has yet to be done to combine those features with immersive AR headsets. We address this issue by developing an interactive AR system featuring Wizard-of-Oz handwriting input, object manipulation, and a custom 3D function plotter. We evaluated our system using a within-subjects study wherein 10 participants compared our system to two other commonly used 3D visualization systems: Geogebra 3D (desktop) and Geogebra 3D Calculator (mobile AR). We found that our AR system significantly surpassed other tools in engagement, achieved comparable ease of use to Geogebra 3D (desktop), and was rated as the most effective in aiding problem-solving, with a strong preference among participants for future use.

CCS Concepts: • **Human-centered computing → Interactive systems and tools**.

Additional Key Words and Phrases: Augmented Reality (AR), Educational Technology, Interactive Learning Tools, Mathematics

1 INTRODUCTION

In this paper, we design and discuss Breaking the Plane, a Meta Quest 3 application that uses OCR input and a 3D function plotter to visualize multivariable functions for improved understanding. For many students, the development of an intuitive understanding of multidimensional mathematical objects, such as multivariate functions and quadric surfaces, has proven a challenge [7]. This is in part due to the inherent difficulty in mentally visualizing abstract mathematical expressions as geometric objects and representing these objects on a 2D surface (ex. paper, blackboard) [9]. Researchers have proposed AR technology as a solution to help students better understand 3D mathematical objects, but classrooms have not widely adopted it yet, despite evidence showing its potential to improve learning outcomes in mathematical education [5].

We tackle this problem of adoption by building an AR headset system with perceived OCR to improve ease of use and engagement in assisting students in the visualization of 3D mathematical objects and subsequently evaluating its effectiveness, relative to other commonly-used visualization techniques, as a better pedagogical tool for visualizing mathematical concepts.

We evaluated how our system (1) engages users more effectively, (2) offers an ease of use that is competitive with existing AR and non-AR systems, and (3) enhances user's self-reported problem-solving abilities. We found that participants using our system felt much more engaged with conceptual mathematical problems, experienced better ease of use than with an existing mobile AR system (Geogebra 3D Calculator), and were likely to use the system again in the future.

2 RELATED WORK

2.1 Applications of AR

Augmented reality (AR) technology has been studied at length as a means of increasing the mathematical creativity of university students, with promising results [8]. Research has demonstrated various AR systems to enhance mathematical learning, contribute to memory retention, and increase motivation to learn mathematics, among other benefits [3, 6, 10, 13]. Consequently, AR technology has been proposed as a solution to help students better understand 3D mathematical objects but has yet to be widely adopted in classrooms, despite having been shown to improve learning outcomes in mathematical education [5]. Experts attribute this difficulty in adoption to a lack of accessible authoring tools and infrastructure for AR teaching materials and user resistance to the use of AR [1, 5, 12].

2.2 Current Systems

Existing AR systems have been developed that aim to address the topics previously mentioned. One such system is Geogebra 3D Calculator Desktop (Geogebra Desktop), a web app that is currently widely adopted in classrooms to help students with plotting and visualizing 3D graphs [2]. However, students have cited not having high confidence when using Geogebra software in a previous study [14]. Another such system is Geogebra 3D Calculator App (Geogebra AR), a mobile app that allows users to visualize 3D graphs in a mixed-reality space. In 2021, Mailizar et al. evaluated the effectiveness of this app in aiding students and found the system to be practical and effective, resulting in students having reported improved visual-spatial understanding after using the tool [11]. However, in this study, the small size of smartphone screens was cited as a system limitation, as users could not see much detail in the augmented graphs, suggesting an lack of ease of use [11].

Taking another approach, researchers at the University of Calgary recently developed an AR interface for mathematical equations adding OCR input [4]. This study found that such a system created a more engaging learning experience for students compared to traditional approaches and, through the use of OCR input, reduced the necessary authoring effort for AR learning materials, allowing for existing textbook resources to be programmatically extended into interactive AR experiences.

2.3 A Novel Approach

Although past approaches have addressed this issue of aiding mathematical understanding through designing AR systems that separately employ OCR and 3D AR, work has yet to be done to combine the two. The combination of an AR system with OCR capabilities has previously been compared to a regular web AR system in studying 2D visualizations of mathematical concepts [11], but it has not been extended to understanding 3D visualizations. Such a synthesis holds value, as an AR system which supports the generation of 3D mathematical visualizations based on print or handwritten material can both take advantage of the unique strengths of AR systems in improving individuals' ability to interpret 3D geometry and the ability of OCR to reduce the friction of information input and content authoring. Thus, we propose a novel system that extends 3D AR visualization tools with OCR performed on print materials and handwriting as a tool to understand multidimensional mathematical objects in a more accessible manner.

3 IMPLEMENTATION

3.1 3D Function Plotter

To visualize mathematical expressions as interactive objects in an augmented reality (AR) scene, our system requires a component which can parse a user-input mathematical function and output a three-dimensional surface representation of the function as a mesh model. Due to the lack of existing open-source resources providing such functionality, we elected to implement this component of our system as a custom script which accepts as input a string of characters to be parsed as a function of two variables (x, y) and generates a procedural mesh that plots the output of the parsed function over a user-specified domain with a variable sampling density. To interpret the text representation of the user's function, we utilize NCalc, a library for mathematical expression evaluation, to calculate the numerical values of the string-formatted function over the input domain.

To constrain the visible bounds of the plotted function and better convey the relative value of the function across its input domain, we wrote a custom GLSL fragment shader that uses the rendered pixel's height in local object coordinates to determine its color using a custom colormap gradient and restrict the surface to a variable height range. Finally, the graph is displayed with labeled axes and a text representation of the plotted function to inform the user of the current input domain and mathematical expression of the visualization.

3.2 On-head Display with Meta Quest 3

Our vision is to enable an intuitive, hands-free experience in which users can simply wear a headset while studying multivariable equations and work alongside a hassle-free aid for mathematical visualization that extends their own mental model. When a user encounters a multivariable equation either in printed text or written text, the headset would automatically detect the equation and render its 3D graph in the user's field of view. Users can then interact with these graphs: rotating, inspecting from various angles, and editing equations to see real-time changes in 3D space. Thus, we developed our final system for the newly released Meta Quest 3 using Unity3D. The Quest 3 is a VR headset, featuring color passthrough, developed by Meta that comes with two controllers to allow for interaction with virtual objects. Via the Quest 3, users can see their physical surroundings while viewing and interacting with virtual graphs in full stereoscopic depth.

3.3 Manipulation of AR Graph

To bring our AR system to basic feature parity with the other visualization systems used in our study, we implemented controls for the graph's input and output domains that allow the user to better manipulate, and hopefully understand, the visualized 3D mathematical surface. To allow the user to move the AR graph about their physical surroundings, our system makes use of the Quest 3's Touch controllers to permit the user to grab the graph and move it around using the secondary hand trigger. In addition, to translate and scale the plotted function within the graph, the user can pan and zoom in/out the input domain (the x and y axes) using the thumbstick and A/B buttons, respectively; these same controls modify the z-axis limits when the user holds the primary trigger. Finally, to allow the user to easily re-center and re-scale the graph to its initial state, one can click the thumbstick to reset all axis parameters to their default values.

3.4 Limitations of Passthrough AR

While the Meta Quest 3 headset's Passthrough layer enables users to view their physical environment when wearing the headset, the Meta API prevents external app developers from accessing, viewing, or storing images from the headset's cameras and sensors of the user's physical environment. Therefore, while our original OCR method was able to make use of camera data from the user's mobile device to allow for direct handwritten input into the mathematical function visualizer, moving our system onto the Meta Quest 3, unfortunately, meant that we could not replicate our original methods while using this hardware, as any OCR technique conducted using the headset alone would require access to the device's camera feed.

3.5 Wizard of Oz Approach

Due to the technical limitations mentioned, we decided to adopt a "Wizard-of-Oz" approach in our studies, in which the user would wear the headset and an operator (a member of our team) would view their streamed live camera feed from the Quest device and manually input the equation to be rendered on the graph. We established a WebSocket connection between our local laptop and the headset to facilitate real-time communication between the two. To this end, we also implemented a command that allows the "Wizard" to broadcast a loading indicator stating "OCR Processing…" to the user, thus disguising operator delay by the appearance of a plausible computation time for OCR and granting the end user a more consistent indicator of the system's state. Overall, this method grants the user the same experience as if our system featured working OCR, has the added benefits of functioning without explicit input by the user for scanning, and is likely to offer a better degree of equation transcription accuracy in user studies.

Previous solutions have focused on allowing users to manually input equations into software for improved visualization and manipulation. We believe that our simulation of dynamic OCR improves a core problem of ease of use with AR-based approaches to mathematical learning by allowing for automatic, hands-free input and more effective manipulation.

4 EVALUATION

4.1 Study Participant Demographics

To evaluate our system, we recruited 10 adult participants (6 males, 4 females), all of whom are Princeton University students who have formerly taken or are currently taking the MAT201 or equivalent Multivariable Calculus course. We recruited most of the participants by sending out emails to student email listservs providing brief information about the study and incentivizing them with a $15 payment in compensation for their participation. This was the recruiting method except for our very last study participant, who we recruited by asking students at the Engineering library if they had taken MAT201 and were interested in participating in a study (since the original study participant mistakenly signed up when they had not taken a Multivariable Calculus course).

4.2 Study Procedure and Data Collection Methodology

For our user study, we employed a within-subject evaluation design in which each participant tests four systems for mathematical visualization. Each session lasted approximately 45 minutes, and our team had a planned-out routine and script to follow to keep each study as similar as possible. After giving a brief introduction and biographical questions, participants underwent four rounds. Each round assigned a quadric surface and system to the user, both of which were selected in independent random sequences to counteract order-dependent effects. The system choices were as follows:

(1) No system/technology, whiteboard and marker only
(2) Geogebra 3D Calculator AR (iOS application)
(3) Geogebra 3D on Desktop (Web Application)
(4) Our newly developed system with the Oculus Quest 3 and OCR handwriting input

4.2.1 Evaluation Study Questions. For each of the four random pairings of the above visualization systems and 3D surface types, each participant was asked a series of questions that would test their ability to make predictions about the relationship between the mathematical function and its geometric and spatial properties. Note that these questions were not graded for 'correctness' – their sole purpose was to prompt the user to (optionally) engage with the provided visualization system, should they find it potentially useful. See Appendix for the questions.

Fig. 1. Our technical systems in the order: Geogebra AR, Geogebra 3D Desktop, Breaking the Plane (our system on Meta Quest 3)

4.2.2 Post-Demonstration Questionnaire. After all four rounds of system demonstration, we asked each study participant to fill out a post-interview questionnaire about their specific experiences with each system and overall feedback on our study. Questions regarding participants' experiences with the four visualization systems types were formatted as 5-point Likert scale responses rating each system's perceived ease-of-use, user engagement, and the participant's willingness to use the system again, to facilitate statistical comparisons of these properties across systems. In addition, the questionnaire required the user to rank the four systems in order of their relative effectiveness in solving the provided problems, and also included 5-point Likert scale questions directly comparing components of our headset AR system, such as handwriting input via OCR and 3D AR visualizations, to those of the mobile AR and desktop systems. Finally, several optional, open-ended questions prompted the user to share their thoughts on the four systems' ease of use, as well as the design and execution of the study.

5 STUDY RESULTS

The study's results indicate that while our AR headset system and Geogebra Desktop are perceived similarly in terms of ease of use, our system significantly outperforms Geogebra AR, No System, and Geogebra Desktop in engagement, with participants showing a marked preference for our system due to its interactive AR features and OCR functionality. Furthermore, the AR headset system was most often ranked as the most effective tool for solving mathematical problems, with participants expressing a strong likelihood of adopting our system in the future due to its user-friendly interface and superior functionality in educational contexts.

5.1 Ease of Use

The t-test results provided in Fig. 2 demonstrate significant variances in ease of use among different systems. One participant pointed to this in their response, "The mobile app was the hardest learning curve to grasp since it was such a small screen and not really able to move the graph," reflecting the lower perceived ease of use for the Geogebra AR and no system. While comparisons between Geogebra Desktop and Geogebra AR, as well as between Geogebra AR and our system, showed statistically significant differences (p-value < 0.05), there was no significant disparity in ease of use between No System and Geogebra AR, or Geogebra Desktop and our system, indicating a comparable ease of use for our system and Geogebra Desktop and a clear user preference for these two over Geogebra AR and the lack of any computer-aid visualization.

5.2 Engagement

The paired t-tests in Fig. 2 indicated clear differences in engagement levels across conditions. Participants demonstrated statistically significantly higher engagement with "Our System" compared to all the other systems: "No System" (t-statistic = -3.667, p-value = 0.008 < 0.05), "Geogebra Desktop" (t-statistic = -5.000, p-value = 0.004), and "Geogebra AR" (t-statistic = -3.057, p-value = 0.022) suggesting a notable increase in engagement when using our headset-based AR system. The 95% confidence intervals for mean engagement scores displayed below not only confirmed the precision of these differences but also underscored the effectiveness of our system in augmenting educational problem-solving engagement.

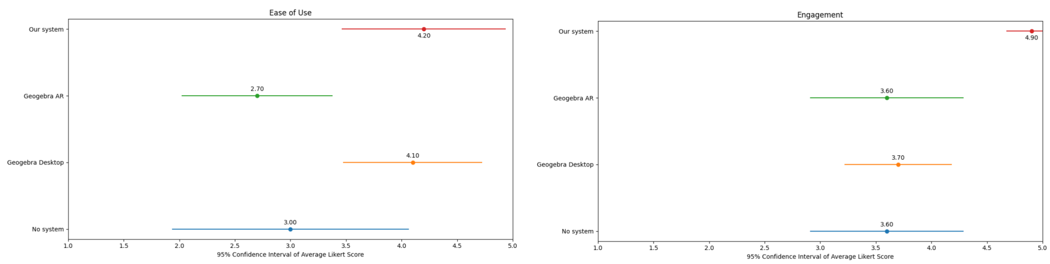

Fig. 2. Graphs comparing user response on the perceived levels of engagement and ease of use with different systems. Likert-scale responses are converted to an integer (1-5) scale and displayed with a 95% confidence interval.

5.3 Effectiveness in Solving Problems

Of the four systems tested, our AR headset system was most frequently ranked as the most effective (6 times) and second most effective (4 times) in aiding mathematical problem-solving during the study. The Geogebra Desktop system was also perceived positively, with several rankings in the top two positions. The other systems were more commonly ranked as less effective, with the Mobile AR System most often ranked third and no system most often ranked as the least effective.

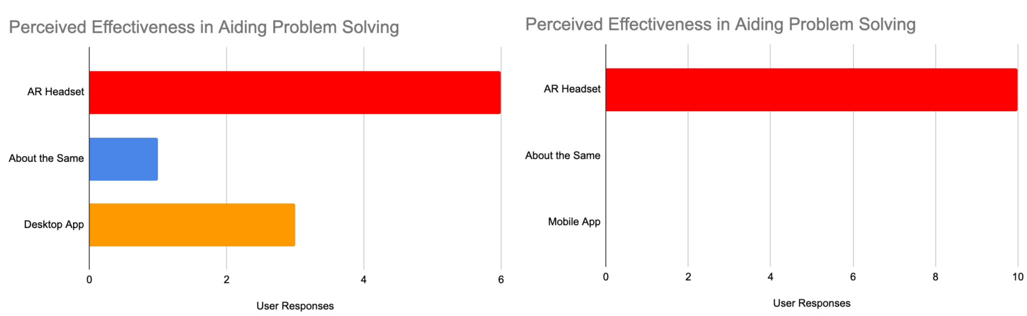

Fig. 3. Direct comparison of our system with Geogebra Desktop, Geogebra Mobile in aiding problem solving

5.4 Satisfaction/Future Adoption of Our System

In the final survey, participants expressed high satisfaction with our AR system, particularly valuing its intuitive graph manipulation and OCR capabilities. This satisfaction was quantified with a high mean score of 4.8 (on a 5-point Likert scale) for the AR headset's features, significantly surpassing the mobile app's mean score of 2.3. Additionally, the likelihood of future use was strong, reflected in a mean score of 4.3 for potential reuse of the AR headset. The OCR feature's contribution to problem-solving was also highly rated, with a mean score of 4.6. These statistics underscore the system's superior functionality and user experience, suggesting its potential for future adoption in educational settings.

6 LIMITATIONS

Our findings are limited by several factors including the use of a Wizard-of-Oz (WoZ) approach for OCR input, a limited sample size of participants from a homogeneous population, and the influence of potential learning effects on our study. The choice of a WoZ solution using manual input for our system's OCR component reduced the applicability of our findings to future systems using fully-functional OCR methods and introduced variability to our evaluation study due to inconsistencies in operator performance: in general, there was significant variance in system response times to new user input, which may influence perceived ease of use and utility. Resulting from our within-subjects design, we also observed the self-reporting of a learning effect within our study wherein several participants noted that they felt the conceptual questions in the final rounds of our system demos to be easier than those in the early rounds, as they had remembered results and visualizations from previous systems. To mitigate this effect, future studies should select conceptual questions and multivariable equations from a larger set of potential options, ensuring that participants do not receive repeated or similar material. Most significantly, the sample size of our study was very limited (n = 10) and was drawn from a small, relatively homogeneous demographic: undergraduate students who had previously taken a college-level multivariate calculus course. To address the utility of AR methods for mathematical pedagogy in a variety of educational contexts, future work should explore the responses to pedagogical AR systems of diverse student populations.

7 DISCUSSION

Our data analysis demonstrates a significant user preference for our system over existing methods for mathematical visualization for the tasks presented, most notably in comparison to Geogebra 3D Calculator AR (Geogebra AR), a prominent existing AR system (Fig. 3). The findings of our study strongly support the hypothesis that the novel features introduced by our system – headset-based AR and handwriting input via OCR – enhance the user experience by significantly improving (relative to Geogebra AR) ease of use, user engagement, and willingness to reuse the AR headset. For example, P2 expressed, "I found it somewhat difficult at first to use the AR headset since I had never used it before, but was able to adapt to it." P9 noted "[the OCR input allowed me] to write down the equations instead of having to learn the syntax for typing them into the desktop or mobile app".

The enhanced engagement provided by our AR system is critical in educational contexts, as the ability to attract and maintain user interest is often correlated with better learning outcomes. In addition, our system and Geogebra Desktop were found to be comparably user-friendly, suggesting that AR technology can be as intuitive as traditional desktop applications in mathematical learning contexts given effective interface design. Nevertheless, some users still preferred the Desktop app, potentially due to prior use and the familiarity of mouse-and-keyboard interfaces. We think there may be a tradeoff between some user's preferences for a system that is already familiar to them and easy to access versus one that is holistically more intuitive but has a slight learning curve. Further research with larger sample sizes and diverse educational contexts would be beneficial to fully understand the implications of AR in education.

8 ETHICS AND ACCESSIBILITY

We are a group of undergraduate students majoring in computer science at Princeton University. Our academic backgrounds and interests, primarily centered around emerging technologies including AR, have undoubtedly influenced the direction and focus of our research. Our enthusiasm for AR technology, especially in educational settings, has shaped our research questions and interpretations, as has our own prior experience as students of university-level mathematics courses. Additionally, we recognize that our perspectives as students at a prestigious university might limit our understanding of the broader implications and applications of AR technology. We have made efforts to mitigate these biases through diverse literature reviews and discussions with experts.

While we are optimistic about the contributions of our research to the field of AR in education, we acknowledge the potential for misuse. For instance, individuals could employ AR technology in ways that compromise privacy. To mitigate these risks, we advocate for the implementation of robust ethical guidelines and regulations governing the use of AR technology; a present example of which is the very camera data restriction imposed by Meta which motivated our Wizard-of-Oz approach. To promote transparency, we support open-sourcing our system code and detailed documentation of our research methodologies and findings.

Our study involved undergraduate participants from Princeton University, none of whom had reported vision, hearing, or haptic disabilities. Additionally, the Quest 3 headset used in our study poses challenges for users with visual impairments or motor control issues. Recognizing this, we propose future research to explore alternative control methods in AR environments, such as voice commands or adaptive controllers.

9 CONCLUSION AND FUTURE WORK

Our research delves into the potential of 3D AR visualizations in enhancing the understanding of multidimensional mathematical concepts. We successfully implement and evaluate an AR headset system equipped with OCR-based handwriting input to facilitate a deeper and more intuitive comprehension of complex mathematical objects. Our findings indicate that our system, compared to mobile AR and traditional flat-screen visualizations, significantly improves user engagement and rivals the ease of use of a desktop-based application, Geogebra Desktop. This suggests a significant stride towards integrating AR technology in educational contexts to address the challenges of visualizing multidimensional mathematical objects.

Key insights from our study reveal that participants not only found our system to be highly engaging due to its interactive AR features but also appreciated the OCR functionality for its ease of input. This level of engagement is crucial in educational settings where user interest is closely linked to learning outcomes. Moreover, despite initial adjustment challenges, users quickly adapted to the system, highlighting its potential for a wider adoption in educational environments. However, our research also encounters certain limitations, including the reliance on a Wizard-of-Oz approach for handwriting input, a study demographic bounded to multivariate calculus students, and a small sample size. These factors limit the generalizability of our findings and suggest the need for further research with diverse populations and practical OCR implementations.

Looking ahead, we envision exploring alternative input methods in AR environments, such as voice commands or gesture recognition, to make our system more accessible to users with different abilities, as well as the potential for multi-user collaboration. In conclusion, our study underlines the transformative potential of AR in educational settings, particularly in mathematical education. Future research should focus on expanding the reach of AR technology, which enhances user engagement and simplifies complex concept visualization, to facilitate greater inclusivity and wider adoption of AR in modern education.

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